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PLENARY SPEAKERS

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Abstracts

Algebraic volumes in geometric tomography

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The talk addresses the problem of characterizing domains or surfaces via the algebraic structure of their volume functions, defined through hyperplane cross-sections. This question, related to Arnold's problem (1987) on algebraically integrable domains, has been the focus of active research in recent years (cf.[1]).

References

[1] M. Agranovsky, J. Boman, A. Koldobsky, V. Vassiliev and V. Yaskin, *Algebraically integrable bodies and related properties of Radon transform*; in: Harmonic Analysis and Convexity, Advances in Analysis and Geometry, **vol. 9**, De Gruyter, (2023), pp. 1–36.

Reconstruction of a Centrally-Symmetric Convex Body by the Orientation-Dependent Chord Length Distribution in R^n

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The problem of convex body reconstruction from random distributions connected with it is the central problem of stochastic geometry and geometrical tomography. Reconstructing a convex body usually means obtaining its support function or its radial function. Averkov and Bianchi [1] proved that the orientation-dependent chord length CDF by all directions can reconstruct any convex domain

in R^2 . However, no universal algorithm is known for that reconstruction. Current work is devoted to the reconstruction of the centrally symmetric convex body from its orientation-dependent chord length cumulative distribution function by restoring the body's width functions in R^n .

References

[1] Averkov, G., Bianchi, G. (2009). Confirmation of Matheron's conjecture on the covariogram of a planar convex body. *Journal of the European Mathematical Society*, 11(6), 1187.

Inversion of the two-data spherical Radon transform from local data

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Hyperplane is a set of non-injectivity of the spherical Radon transform (SRT) in $\mathbf{R}^d$. It is possible to reconstruct a function compactly supported on one side of a hyperplane using SRT over spheres centered on the hyperplane. The problem arises: to find an additional condition for the reconstruction of $f \in C(\mathbf{R}^3)$ (the support can be non-compact) using SRT over spheres centered on a plane. An additional condition is found, which is a weighted SRT (to reconstruct an odd function with respect to the hyperplane), and the injectivity of the so-called two data spherical Radon transform is considered. The transform consists of the classical SRT and the weighted SRT.

Given a continuous function $f \in C(\mathbf{R}^3)$ for $(P, t) \in \{z = 0\} \times (0, \infty)$ we define

$$Wf(P, t) = \frac{1}{4\pi} \int_{\mathbf{S}^2} \cos \theta f(P + t\omega) d\omega, \quad (1)$$

here $(\theta, \varphi) = \omega \in \mathbf{S}^2$ is the spherical coordinates of ω ($\theta \in [0, \pi]$ is the polar angle measured from the z -axis) and we integrate with respect to the surface Lebesgue measure on $\mathbf{S}^2$.

Let $D \subset \{z = 0\}$ be a planar domain (an open subset). The transform (two data spherical Radon transform)

$$f \rightarrow (Mf(P, t), Wf(P, t)) \text{ for } (P, t) \in D \times (0, \infty) \quad (2)$$

is invertible in $C^1(\mathbf{R}^3)$. Here $Mf(P, t)$ is the spherical mean.

An inversion formula of the transform that uses the local data of the spherical integrals to reconstruct the unknown function is found. Such inversions are the mathematical base of modern modalities of imaging, such as Thermo and photoacoustic tomography and radar imaging, and have theoretical significance in many areas of mathematics.

References

- [1] R. Aramyan, *Recovering a function from spherical means in 3D using local data*. Inverse Problems and Imaging, 2024, 18(3): 690-707.
- [2] R. Aramyan, *Inversion of the two-data spherical Radon transform with the centers on a plane*, Jour. of Mathematical Analysis and Applications, Volume 548, 2025

Reconstruction of Geometric Moments of a Convex Body from the Distribution of Directed Chords

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For the reconstruction of an object Stochastic tomography uses the distribution of X-rays (projections), which can be estimated from the sample. Using random samples makes it possible to simplify the calculation since mathematical statistics methods can be used to estimate the geometric characteristics.

Let B be a convex body in $\mathbb{R}^n$. We consider a random directed line g (intersecting B) with normalized invariant measure and by $\overrightarrow{\chi(g)}$ we denote the oriented chord

$D \cap g$, the vector specified by the direction of g and its length χ . Thus we induce the probability distribution in $\mathbb{R}^n$.

Definition The probability distribution of $\overrightarrow{\chi(g)} = D \cap g$ is called the joint distribution of the oriented random chords of the body.

Statement of the problem: Does the distribution of the oriented random chords of a convex body identify the body up to translations and reflections?

Note that the problems of reconstruction of a convex body from the random chord length distribution and the orientation-dependent chord length distribution (one dimensional distributions) were considered in ([2]).

In $\mathbb{R}^2$ the answer is positive (see [1]). Also in [1] was presented a novel approach to reconstruct a planar convex domain from its random oriented chord distribution by recovering the real moments of the domain.

In this report we obtain a relations that allows us to recover the geometric moments of $B \subset \mathbb{R}^3$ from the oriented random chord distribution of B . Next, using mathematical statistics methods we estimate the geometric moments of B by random sample of lines intersecting B .

References

- [1] R.H. Aramyan, R.M. Mnatsakanov, E. Aramyan, L. Apinyan, F. Jafari, "Reconstruction of a planar centrally symmetric convex domain by random chord distribution", Lobachevskii Journal of Mathematics, 2024, Vol. 45, No. 12
- [2] G. Averkov and G. Bianchi, "Confirmation of Matheron's conjecture on the covariogram of a planar convex body," J. Eur.Math. Soc. 11, 1187–1202 (2009).

Wave dualism as a measure of the complexity of the dynamics of the initial system¹

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The report describes the procedure for representing the trajectories of the initial dynamic system by soliton solutions (traveling wave type solutions) of a dual

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system. The complex dynamics of the initial system in the dual representation of the corresponding soliton solutions is identified by a number of parameters (it turns out to be observable). In the case of specifying the initial system as a system of ordinary differential equations in a dual system, trivial systems of soliton solutions correspond to the initial trajectories. If the initial system is defined by a system of pointwise functional differential equations, then the corresponding soliton solutions of the dual system have a characteristic defined by a group of linear homeomorphisms that determine the deviations of the argument in the initial system.

References

- [1] L.A. Beklaryan, A.L. Beklaryan, *Dualism in the theory of soliton solutions I*. Computational Mathematics and Mathematical Physics **64**(7) (2024), pp. 1472–1490.
- [2] L.A. Beklaryan, *Dualism in the theory of soliton solutions II*. Computational Mathematics and Mathematical Physics **64**(11) (2024), pp. 2588–2610.

Mixed volumes of convex hulls of random processes

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Let $K_1, K_2, \dots, K_s$ be convex bodies in $\mathbb{R}^d$. Minkowski showed that d -dimensional volume $\text{Vol}_d(\lambda_1 K_1 + \lambda_2 K_2 + \dots + \lambda_s K_s)$ with $\lambda_1, \lambda_2, \dots, \lambda_s \geq 0$ is homogeneous polynomial of degree d :

$$\text{Vol}_d(\lambda_1 K_1 + \lambda_2 K_2 + \dots + \lambda_s K_s) = \sum_{i_1=1}^s \dots \sum_{i_d=1}^s \lambda_{i_1} \dots \lambda_{i_d} V_d(K_{i_1}, \dots, K_{i_d}),$$

where functions $V_d(K_{i_1}, \dots, K_{i_d})$ are symmetric and called mixed volumes.

We consider convex hulls of independent random walks with exchangeable increments and calculate the mathematical expectation of their mixed volumes. As a corollary, we obtain a similar result for independent symmetric stable Lévy processes.

On geometric identifiability conditions in problems of source localization on the plane

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In a recent series of papers [1–6], the problem of localization (estimation of the position) of a source emitting Poissonian signals registered by K detectors on the plane was studied in many different setups.

First, the information about the position of the source can be contained either in the amplitude of the received signal (this case is called RSS, or received signal strength, in the engineering literature), or in the moment of arrival of the signal to the detector. In the latter case, the moment of emission of the signal can be known (case called TOA: time of arrival) or unknown (TDOA: time difference of arrival). Moreover, three different types of regularity of the front of the signal (smooth, cusp and change-point) were studied. Further extensions are the case of two sources and, in the case of RSS, the unknown initial amplitude.

Second, two approaches to localization were considered and the asymptotic properties of the estimators were studied for each of them. The first approach, known as “direct positioning”, consists in collecting the data from all detectors, calculating the likelihood ratio, and then deducing maximum likelihood and/or Bayesian estimators. Another approach is the “two-step positioning”. In the TOA/TDOA cases it can be described as follows: first, on the base of the observations registered by each detector, the moment of arrival of the signal to the detector is estimated, and then the obtained estimators are transmitted to the center of data treatment, where, using these values, the final estimator is constructed (in general using the least squares method).

In this talk we present some of the results of these papers with a special focus on the identifiability conditions. For the cases of a unique source and either TOA, or RSS with known amplitude, it is well known that the system will be identifiable if there exist at least three non-aligned detectors. However, quite intriguingly, except for these simplest cases, the identifiability conditions seem not to be present in the (rich but mainly engineering) literature on the subject.

References

- [1] O.V. Chernoyarov, Yu.A. Kutoyants, *Poisson source localization on the plane: the smooth case*. *Metrika* **83** (2020), pp. 411–435.
- [2] C. Farinetto, Yu.A. Kutoyants, A. Top, *Poisson source localization on the plane: change-point case*. *Annals of the Institute of Statistical Mathematics* **72** (2020), pp. 675–698.
- [3] O.V. Chernoyarov, S. Dachian, Yu.A. Kutoyants, *Poisson source localization on the plane: cusp case*. *Annals of the Institute of Statistical Mathematics* **72** (2020), pp. 1137–1157.
- [4] O.V. Chernoyarov, S. Dachian, C. Farinetto, Yu.A. Kutoyants, *Estimation of the position and time of emission of a source*. *Statistical Inference for Stochastic Processes* **25** (2022), pp. 61–82.
- [5] O.V. Chernoyarov, S. Dachian, C. Farinetto, Yu.A. Kutoyants, *Localization of two radioactive sources on the plane*. *Statistical Inference for Stochastic Processes* **27** (2024), pp. 1–23.
- [6] O.V. Chernoyarov, S. Dachian, Yu.A. Kutoyants, *Localization of moving Poisson source on the plane*. *Annals of the Institute of Statistical Mathematics* (2025), published online.

Parallelized Midpoint Randomization for Langevin Monte Carlo

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We study the problem of sampling from a target probability density function in frameworks where parallel evaluations of the log-density gradient are feasible. Focusing on smooth and strongly log-concave densities, we revisit the parallelized randomized midpoint method and investigate its properties using recently developed techniques for analyzing its sequential version. Through these techniques, we derive upper bounds on the Wasserstein distance between sampling and target densities. These bounds quantify the substantial runtime improvements achieved through parallel processing. This is a joint work with Lu Yu (City University of Hong Kong).

Point Process Approach to the Winner Problem

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In our paper [1], we considered the asymptotic behavior of the Argmaximum of a large number of independent random variables (r.v.'s). In [1], we also called this the winner problem. If the r.v.'s are identically distributed, the answer is obvious: from the very beginning, the distribution of the Argmaximum is uniform, while for non-identically r.v.'s, the problem turned out to be non-trivial. Clearly, for the result to be substantial, the tail distributions should have similar in a sense character. Since the distribution of an Argmaximum is invariant under a strictly increasing transformation of the r.v.'s, we may suppose from the very beginning that the mentioned tails satisfy the condition of regular variation. Since the last condition is strongly connected with the convergence of empiric point processes (see, for example Resnick [2], Proposition 3.21), it could be natural to use this connection for the analysis of the limiting behavior of the Argmaximum distribution. To this end, we first prove a theorem (Th.1) on the convergence of point processes for a triangular array of non-identically distributed r.v.'s. This theorem generalizes above mentioned Resnick's theorem. Next, we derive from Theorem 1 a number of corollaries on the behavior of the Maximum, Argmaximum, and step processes. Our previous result about argmaximum is also partially covered by this theorem.

The talk is based on joint work with Vladimir Rotar (San Diego, UCSD).

References

- [1] Davydov Yu., Rotar, V.I., The Distribution of Argmaximum or a Winner Problem, Statistics and Probability Letter, V 211, 110152, (2024); also (2023) <https://arxiv.org/2305.05967>.
- [2] S. I. Resnick. Extreme Values, Regular Variation and Point Processes, Springer, Berlin, 1987.

Mixed volumes and Gaussian processes on convex compact sets

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Let K be a convex compact subset of $\mathbb{R}^d$. Each such compact set K has characteristics that do not depend on the dimension of the ambient space d , but depend only on the intrinsic geometry of K . These characteristics are called *intrinsic volumes* $V_k(K)$, $k = 0, 1, \dots, d$, and they are defined as the coefficients in the Steiner formula. Steiner proved that the volume of the λ -neighborhood of K is represented by a polynomial in λ with coefficients $V_k(K)$ (where the normalization is chosen in a special way). The intrinsic volumes of a set do not depend on the dimension d of the ambient space $\mathbb{R}^d$, which enabled Sudakov and Chevet to extend the concept to infinite-dimensional convex sets.

Later, Sudakov [1] and Tsirelson [2] discovered a deep connection between the intrinsic volumes of convex compact sets and Gaussian processes on these sets.

A generalization of intrinsic volumes are the so-called *mixed volumes*, which are defined in a similar way, namely, as the coefficients in the Minkowski's formula for the volume of a Minkowski sum of an arbitrary number of finite-dimensional compact sets.

This talk will present a generalization of Sudakov-Tsirelson theorem on the Gaussian representation of intrinsic volumes [2] to the mixed volumes of the infinite-dimensional convex compact sets. Using the obtained result, we calculate the mixed volume of the closed convex hulls of the two orthogonal Wiener spirals.

References

- [1] V. N. Sudakov, *Geometric problems in the theory of infinite-dimensional probability distributions*. Proc. Steklov Inst. Math. **141** (1979), pp. 1–178.
- [2] B. S. Tsirelson, *Geometrical approach to the maximum likelihood estimation for infinite-dimensional Gaussian location. II*. Teor. Veroyatnost. i Primenen. **30**(4) (1985), pp. 772–779.
- [3] H. Minkowski, *Theorie der konvexen Körper, insbesondere Begründung ihres Oberflächenbegriffs*. Gesammelte Abhandlungen. **2** (1911), pp. 131–229.

- [4] R. Schneider, *Convex bodies: the Brunn-Minkowski theory*. Cambridge: Cambridge University Press (2014).
- [5] R. A. Vitale, *Convex bodies and Gaussian processes*. Image Anal. Stereol. **29(1)** (2010), pp. 13–18.

Realizability of point processes via the Kirkwood-Salsburg equations

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Given a family of functions $\rho^{(n)}$, $1 \leq n \leq M$ it is a natural question if there is a point process such that the given functions are the first M correlation functions of this process. This is known as the *realizability problem* of point processes.

A special case of this problem is when these functions can be written as the negative exponential of some Hamiltonian H . For the case that H only consists of a pair potential u the functions $\rho^{(n)}$ coincide with the Kirkwood superposition approximation from statistical physics. In this case existence of the so-called *Kirkwood closure process* was first shown by Ambartzumian and Sukiasian under the restriction that u is non-negative and $\rho^{(1)}$ is sufficiently small. Kuna, Lebowitz and Speer generalized this result to the case that u is a locally stable and regular pair potential.

In this talk, it is shown that it suffices for u to be stable and regular to ensure the existence of the Kirkwood closure process. Furthermore, for locally stable u it is proved that the Kirkwood closure process is Gibbs and that the kernel of the GNZ-equation satisfies a Kirkwood-Salsburg type equation. Lastly, we mention how this ansatz can be generalized to multi-body Hamiltonians.

References

- [1] R.V. Ambartzumian and H.S. Sukiasian: Inclusion-exclusion and point processes, Acta Appl. Math. **22** (1991), pp. 15–31.

- [2] J.G. Kirkwood and E.M. Boggs: The radial distribution function in liquids, J. Chem. Phys. **10** (1942), pp. 394–402.
- [3] T. Kuna, J.L. Lebowitz and E.R. Speer: Realizability of point processes, J. Stat. Phys. **129** (2007), pp. 417–439.
- [4] V.I. Skrypnik: Solutions of the Kirkwood-Salsburg equation for particles with finite-range nonpairwise repulsion, Ukr. Mat. J. **60** (2008), pp. 1329–1334.

On sample function continuity of Gaussian Process in Hilbert Space via Steiner entire function

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Let K be a convex compact set in $\mathbb{R}^d$. The classical Steiner formula expresses the volume of the r -neighborhood of K as a polynomial in r . The renormalized coefficients of this polynomial are called intrinsic volumes and are denoted by $V_j(K)$. They generalize geometric quantities such as volume, surface area, and mean width; moreover, they are intrinsic, meaning that they depend only on the internal geometry of the body and not on the dimension of the ambient space in which the body is embedded. This property allows one to define analogous characteristics for compact sets in a separable Hilbert space.

There is no direct analogue of the Steiner formula in Hilbert space. Instead, we introduce Steiner entire function, which is also a generating function for the sequence of intrinsic volumes:

$$f_K(z) = \sum_{j \geq 0} V_j(K) z^j.$$

Vitale [1] related the limits of the quantities

$$m_j = \frac{(j+1)V_{j+1}(K)}{V_j(K)}$$

to the sample continuity/boundedness of a Gaussian process defined on the compact set K . Gao and Vitale [2] initiated the study of the asymptotic behavior of $V_j(K)$ and m_j for GB -sets K in a Hilbert space. They computed these quantities explicitly for the closed convex hull of the Wiener spiral and observed a specific decay rate $m_j = O(j^{-1/2})$ in that case. Based on this and related evidence, Gao and Vitale formulated the following conjecture:

Conjecture. For every convex GB -compact set K in a separable Hilbert space, either

$$\lim_{j \rightarrow \infty} m_j(K) > 0,$$

or

$$m_j(K) = O(j^{-1/2}) \quad (j \rightarrow \infty).$$

In other words, the sequence $m_j(K)$ cannot decay to zero at a rate slower than $j^{-1/2}$.

In this talk, we present a disproof of this conjecture, which is based on considering the generating function for intrinsic volumes as an entire function. We also provide a criterion for the continuity of sample paths of the process in terms of the order and type of this entire function.

The talk is based on joint work [3] with Maria Dospolova and Dmitry Zaporozhets.

References

- [1] R. A. Vitale, *Intrinsic Volumes and Gaussian Processes*, Advances in Applied Probability, vol. 33, no. 2, pp. 354–364, 2001.
- [2] F. Gao and R. A. Vitale, *Intrinsic volumes of the Brownian motion body*, Discrete & Computational Geometry, vol. 26, no. 1, pp. 41–50, 2001.
- [3] M. Dospolova, M. Germanskov, and D. Zaporozhets, *On Steiner entire function*, arXiv:2507.11626, 2025.

Statistical problems for Smoluchowski processes

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Suppose that particles are randomly distributed in $\mathbb{R}^d$, and they are subject to identical stochastic motion independently of each other. The Smoluchowski process describes fluctuations of the number of particles in an observation region over time. This talk discusses probabilistic properties of the Smoluchowski processes and considers related statistical problems. We revisit probabilistic properties of the Smoluchowski processes in a unified and principled way: explicit formulas for generating functionals and moments are derived, conditions for stationarity and Gaussian approximation are discussed, and relations to other stochastic models are highlighted. The second part of the talk deals with statistics of the Smoluchowski processes. We consider two different models of the particle displacement process: the undeviated uniform motion (when a particle moves with random constant velocity along a straight line) and the Brownian motion displacement. In the setting of the undeviated uniform motion we study the problems of estimating the mean speed and the speed distribution, while for the Brownian displacement model the problem of estimating the diffusion coefficient is considered. We demonstrate how the emerging statistical inverse problems are influenced by the geometry of the observation region and the characteristics of the displacement model. In all settings we develop estimators with provable accuracy guarantees. The talk is based on joint work with R. Jacobovic.

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Vertex degrees behavior in β - and β' -Delaunay graph

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The β -Delaunay and β' -Delaunay triangulations are tessellation models generalizing classical Poisson-Delaunay triangulation on the Euclidean space. Both were introduced recently by Gusakova, Kabluchko, and Thäle in a series of papers [1-4]. We present various results on the degrees of the vertex-edge graphs of these models, including bounds on the distributions of typical degrees and (in one of those models) a concentration phenomenon for the maximal degree in a growing window.

The talk is based on a joint work with Gilles Bonnet that is currently available on arXiv [5].

References

- [1] A. Gusakova, Z. Kabluchko, and C. Thäle., *The β -Delaunay tessellation: Description of the model and geometry of typical cells*. Advances in Applied Probability **54.4** (2022), pp. 1252–1290.
- [2] A. Gusakova, Z. Kabluchko, and C. Thäle., *The β -Delaunay tessellation II: the Gaussian limit tessellation*. Electronic Journal of Probability **27** (2022), pp. 1–33.
- [3] A. Gusakova, Z. Kabluchko, and C. Thäle., *The β -Delaunay tessellation III: Kendall's problem and limit theorems in high dimensions*. ALEA **19.1** (2022), pp. 23–50.
- [4] A. Gusakova, Z. Kabluchko, and C. Thäle., *The β -Delaunay tessellation IV: Mixing properties and central limit theorems*. Stochastics and Dynamics **23.3** (2023).
- [5] G. Bonnet, J. Gordon, *Degrees in the β - and β' -Delaunay graphs* arXiv:2503.24024v1

Distribution of intersection points of n lines within a planar body

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Let $\mathbf{R}^2$ be the 2-dimensional Euclidean space and $\mathbf{D}$ be a body $D \subset \mathbf{R}^2$. Let $n \geq 2$ random lines intersect a body $\mathbf{D}$. Consider probabilities p_{nk} , $k = 0, 1, \dots, \frac{n(n-1)}{2}$, that n random lines produce exactly k intersection points inside $\mathbf{D}$. The objective is finding p_{nk} through geometric invariants of $\mathbf{D}$.

The intersection probabilities of n random lines meeting a planar body is a classical object in stochastic geometry, with a dominant geometric flavour. For a body $\mathbf{D} \subset \mathbb{R}^2$, we consider N_n , the number of intersection points of n random lines in $\mathbf{D}$, given that all n lines meet $\mathbf{D}$. Let $p_{nk} = \mathbb{P}(N_n = k)$, $S(\mathbf{D})$ and $|\partial\mathbf{D}|$ are the area and the perimeter of $\mathbf{D}$, respectively. The computation of p_{3k} requires more invariants of $\mathbf{D}$ besides the area and the perimeter. These are suggested in [1, Chapter 4] to be

$$\mathbf{I}_2 = \int_{g \cap \mathbf{D} \neq \emptyset} |\chi(g)|^2 dg, \quad \mathbf{U} = \iint_{g_1 \cap g_2 \in \mathbf{D}} u(g_1, g_2) dg_1 dg_2.$$

where $\chi(g) = g \cap \mathbf{D}$ is the chord in $\mathbf{D}$ produced by the line g , $|\chi(g)|$ denotes its length, and $u(g_1, g_2)$ denotes the perimeter of the convex quadrilateral whose vertices are the points of intersection of the lines g_1 and g_2 with the boundary $\partial\mathbf{D}$. The measure element dg is interpreted as $dg = dp d\varphi$, where dp is the one-dimensional Lebesgue measure and $d\varphi$ is the uniform measure in the unit circle.

Let

$$v_{ij} = \begin{cases} 1, & \text{if } g_1 \cap g_2 \in \mathbf{D}, \\ 0, & \text{otherwise,} \end{cases} \quad (3)$$

and $\mathbf{E}N_n$ be the expectation of N_n . Using the notation (3), we can write

$$N_n = \sum_{i < j}^n v_{ij}. \quad (4)$$

We can use (3) and (4) to find probabilities p_{nk} .

Proposition 1 *From system of equations*

$$\sum_{j=1}^{\frac{n(n-1)}{2}} j^m p_{nj} = \mathbf{E} \left(\sum_{i < j}^{\frac{n(n-1)}{2}} v_{ij} \right)^m, m = 0, 1, 2, \dots, \frac{n(n-1)}{2} \quad (5)$$

we can find the probabilities p_{nj} , where $j = 0, 1, 2, \dots, \frac{n(n-1)}{2}$ and $n = 2, 3, \dots$.

For any $n = 2, 3, \dots$ there is the Python code that we can use to find the solution to the system of equations (5).

Consider the case $n = 4$ (see [2]). Let us assign

$$\begin{aligned} e_1 &= E[v_{12}], e_2 = E[v_{12}v_{13}], e_3 = E[v_{12}v_{13}v_{14}], e_4 = E[v_{12}v_{13}v_{14}v_{23}], \\ e_5 &= E[v_{12}v_{13}v_{23}], e_6 = E[v_{12}v_{13}v_{14}v_{23}v_{24}], e_7 = E[v_{12}v_{13}v_{14}v_{23}v_{24}v_{34}], \\ e_8 &= E[v_{12}v_{13}v_{24}], e_9 = E[v_{12}v_{34}], e_{10} = E[v_{12}v_{13}v_{24}v_{34}], \end{aligned}$$

solution will be

$$\begin{aligned} p_{40} &= 1 - 6e_1 + 12e_2 - 4e_3 + 12e_4 - 4e_5 - 6e_6 + e_7 - 12e_8 + 3e_9 + 3e_{10}, \\ p_{41} &= 6e_1 - 24e_2 + 12e_3 - 48e_4 + 12e_5 + 30e_6 - 6e_7 + 36e_8 - 6e_9 - 12e_{10}, \\ p_{42} &= 12e_2 - 12e_3 + 72e_4 - 12e_5 - 60e_6 + 15e_7 - 36e_8 + 3e_9 + 18e_{10}, \\ p_{43} &= 4e_3 - 48e_4 + 4e_5 + 12e_8 - 20e_7 + 60e_6 - 12e_{10}, \\ p_{44} &= 12e_4 - 30e_6 + 15e_7 + 3e_{10}, p_{45} = 6e_6 - 6e_7, p_{46} = e_7, \end{aligned}$$

and all expectations we can calculate using Integral Geometry methods. In the following, you can find the results of a few of them:

$$e_1 = \frac{2\pi S(\mathbf{D})}{|\partial\mathbf{D}|^2}, e_2 = \frac{4\mathbf{I}_2}{|\partial\mathbf{D}|^3}, e_5 = \frac{8\mathbf{I}_2 - \mathbf{U}}{|\partial\mathbf{D}|^3}, e_9 = \frac{4\pi^2 S^2(\mathbf{D})}{|\partial\mathbf{D}|^4}, e_3 = \frac{24S^2(\mathbf{D})}{|\partial\mathbf{D}|^4}.$$

References

- [1] L.A. Santaló, *Integral Geometry and Geometric Probability*. Cambridge University Press (2004).
- [2] D. Martirosyan and V. Ohanyan, *On intersection probabilities of four lines inside a planar convex domain*. Journal of Applied Probability **59** (2022), pp. 1–24

Normal approximation of Poisson functionals of column-type dependence

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We study Poisson functionals of column-type dependence, i.e., local behavior in the first k directions and allow non-local, yet stabilizing behavior in the remaining $d - k$ directions. The main result will be a quantitative normal approximation for sequences of such functionals over growing regions. The difficulty is to control general bounds from Malliavin-Stein calculus in the presence of column-type dependencies.

We then apply our results to two guiding examples. First, the crossing number of projected random geometric graphs. Second, functionals such as inversion count and tree-realization of the persistence diagram based on Poisson tree models. We also discuss lower bounds on the variance.

This talk is based on joint work with Hanna Döring, Adélie Garin, and Nikolaj N. Lundbye.

Finding the Area, Perimeter, Distributions for Poisson Line Processes in the Plane

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We present a unified kinetic method to derive the full (unconditional) distribution functions and the *joint law* of the *area* S and *perimeter* P of planar cells generated by a Poisson line tessellation and by the Poisson Voronoi diagram. We

observe a random cell through a moving secant line and track only a few local variables (segment length, two endpoint angles, and the running S or P). This yields kinetic and Markov equations for line-based densities together with exact relations that lift them to spatial distributions. A Laplace transform reduces the perimeter equation to a Riccati type ODE, giving explicit formulas; the area equation leads to integral representations (existence ensured by exponential decay). Because the line-transition probabilities coincide for Poisson lines and Voronoi cells, one system of equations covers both models and produces a global (**total**) distribution that aggregates over all polygon types at once, avoiding enumeration by side count. Beyond these two models, the same kinetic line-scan framework applies often with minor changes to other planar integral-geometry problems in which the target functional accumulates along line crossings (e.g., Crofton type functionals, chord-length statistics), providing a reusable route to full distributions without polygon enumeration.

Our scope does not complement P. Calka's results it is different. Calka provides formulas for each fixed side count $N = n$; here we provide the complete (unconditional) laws of S , P , and (S, P) by aggregating over all n in a single kinetic-transform framework, simultaneously for Poisson lines and Poisson Voronoi cells.

The talk is based on joint work with Mehdi Golafshan.

References

- [1] P. Calka, *An explicit expression for the distribution of the number of sides of the typical Poisson Voronoi cell*. Advances in Applied Probability **35** (2003), pp. 863–870.
- [2] P. Calka, *Precise formulae for the distributions of the principal geometric characteristics of the typical cells of a two-dimensional Poisson Voronoi tessellation and a Poisson line process*. Advances in Applied Probability **35** (2016), pp. 551–562.
- [3] A.Ya. Kanel-Belov, M. Golafshan, S.G. Malev, and R.P. Yavich, *Finding the Area and Perimeter Distributions for Flat Poisson Processes of a Straight Line and Voronoi Diagrams*. Doklady Mathematics (2024).
- [4] R.E. Miles, *The random division of space*. Advances in Applied Probability **4** (1972), pp. 243–266.
- [5] R.E. Miles, *Poisson flats in Euclidean spaces*. Advances in Applied Probability **1** (1969), pp. 211–237.

Gibbs scheme in the theory of random field

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In the second half of the last century, an impressive process of further mathematization of statistical physics began. The result was a broad and deep theory often called mathematical statistical physics. The basis of this theory is the concept of a Gibbs random field with a given interaction potential, introduced by R. Dobrushin [1] and later independently by O. Lanford and D. Ruelle [2].

Mathematical statistical physics can be considered from two points of view — physical and probabilistic. In the course of its creation, the DLR-definition of the Gibbs random field was taken as a basis, which predetermined the dominance of the physical point of view in this process. As a result, mathematical statistical physics (despite the rigour of its results) acquired the structure inherent in physical theories.

Wherein, the probabilistic point of view was not properly developed. The reason is clear: at that time, a purely probabilistic definition of the Gibbs random field based on its intrinsic characteristics, namely, the properties of its finite-dimensional distributions, was not formulated. It should be noted that in the absence of such a definition, many problems arising in mathematical statistical physics (being, in fact, probabilistic) forcedly were solved on the basis of the DLR-definition. The latter often led to rather cumbersome constructions.

The first attempt to give a purely probabilistic definition of the Gibbs random field was made in [3]. However, the proposed definition had some limitations.

In this talk, an improved version of the purely probabilistic definition (or simply Pr-definition) of the Gibbs random field will be presented. According to it, a positive random field is called Gibbsian if its conditional distributions are positive and uniformly continuous with respect to almost all boundary conditions. The Pr-definition reveals the probabilistic meaning of the class of Gibbs random fields as a uniform extension of the class of Markov random fields and allows giving its structure the form accepted in the theory of random processes.

During the talk, the outlines of the (probabilistic) theory of Gibbs random fields will be given. It will be shown that the results of mathematical statistical physics

are embedded in the developing theory of Gibbs random fields as its important (if not the most important) part. Also, the range of those problems that are more natural to solve using suggested PR-definition will be described.

The talk is based on the joint work [4] with Boris S. Nahapetian, Institute of Mathematics, NAS RA. The work was supported by the Science Committee of the Republic of Armenia in the frames of the research project 21AG-1A045.

References

- [1] R.L. Dobrushin, *Gibbs random fields for lattice systems with pair-wise interaction*. Funct. Anal. and Appl. **2** (1968), pp. 292–301.
- [2] O.E. Lanford and D. Ruelle, *Observables at infinity and states with short range correlations in statistical mechanics*. Commun. Math. Phys. **13** (1969), pp. 194–215.
- [3] S. Dachian and B.S. Nahapetian, *On Gibbsiannes of random fields*. Markov Process. Relat. Fields **15** (2009), pp. 81–104.
- [4] L.A. Khachatryan and B.S. Nahapetian, *Gibbs scheme in the theory of random fields*. Ann. Henri Poincaré (2025), 27 pp.

Stochastic Finite-Difference and Subgradient Methods for Nondifferentiable Quasiconvex Optimization: Convergence Rate

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This article examines finite-difference stochastic methods for the minimization of non-differentiable quasi convex functions. It addresses the convergence of theses methods and, in the case of strongly quasi convex functions, establishes explicit convergence rates. The article demonstrates that the known convergence results for the stochastic subgradient methods remain valid when the exact subgradients

are replaced by their approximating stochastic finite-difference schemes.

References

- [1] F. Lara. *On strongly quasi convex functions: existence results and proximal point algorithm* JOTA **192** (2022), pp. 891–911
- [2] M. Jovanovic. *A note a strongly convex and quasi convex functions* Math Note **60** (1996), pp. 584–585
- [3] Yu E. Nesterov, V. Spokoiny. *Random Gradient-Free Minimization of Convex Function* Foundations of Computational Mathematics **17** (2017), pp. 527–566

Tube formula for spherically contoured random fields with subexponential marginals

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It is widely known that the tube method, or equivalently the Euler characteristic heuristic, is an integral-geometric approach that provides highly accurate approximations for the tail probability that the supremum of a smooth Gaussian random field exceeds a threshold value c . The relative approximation error $\Delta(c)$ decays exponentially in c as c tends to infinity. On the other hand, little is known about non-Gaussian random fields.

In this talk, we obtain the approximation error of the tube method applied to the canonical isotropic random fields on a unit sphere defined by $u \mapsto \langle u, \xi \rangle$, $u \in M \subset \mathbb{S}^{n-1}$, where ξ is a spherically contoured random vector. These random fields have statistical applications in multiple testing and simultaneous regression inference when the unknown variance is estimated. The decay rate of the relative error $\Delta(c)$ depends on the tail of the distribution of $\|\xi\|^2$ and the critical radius of the index set M . If this distribution is subexponential but not regularly varying, $\Delta(c) \rightarrow 0$ as $c \rightarrow \infty$. However, in the regularly varying case, $\Delta(c)$ does not vanish and hence is not negligible. To address this limitation, we provide simple upper

and lower bounds for $\Delta(c)$ and for the tube formula itself. Numerical studies are conducted to assess the accuracy of the asymptotic approximation.

The talk is based on joint work with E. Spodarev [1]

Reference

- [1] S. Kuriki and E. Spodarev (2025), arXiv:2507.11154 [math.PR]

Hidden Markov Processes and Adaptive Filtering

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We present a review of the results on adaptive filtering for partially observed systems depending on unknown parameters presented in [1]. Adaptive filters are proposed for a wide variety of models: Gaussian and conditionally Gaussian linear models of diffusion processes; some nonlinear models; telegraph signals in white Gaussian noise (all in continuous time); and autoregressive processes observed in white noise (discrete time). The properties of the estimators and adaptive filters are described in the asymptotics of small noise or large samples. The parameter estimators and adaptive filters have a recursive structure which makes their numerical realization relatively simple. The question of the asymptotic efficiency of the adaptive filters is also discussed.

It is shown how to construct Le Cam's One-step MLE for all these models and how this estimator can be transformed into an asymptotically efficient estimator-process which has a recursive structure.

Several applications of the developed methods are also considered. The problems are: estimation of a signal in noise, identification of a security price process, change point problems for partially observed systems, approximation of the solution of BSDEs and the localization of fixed and moving sources on the plane

by observations of random signals registered by K detectors in the presence of noise. The proposed solutions for the last model are related with the different geometrical problems.

References

- [1] Y.A. Kutoyants, *Hidden Markov Processes and Adaptive Filtering*. Springer, Chem (2025).

Poisson Telecom processes: limit theorems and large deviations

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Two types of Telecom processes introduced by I.Kaj and M.Taqqu appear as the limit processes for the workload of a service system with infinite source input (e.g., a telecom system model) – along with more classical objects such as Wiener process, fractional Brownian motion, stable Lévy process. It is therefore interesting to study their basic properties.

In this talk we explore one of these processes – a Poisson Telecom process which admits an integral representation via Poisson random point measure in $\mathbb{R}^3$. We are interested in large deviation probabilities of the process. It turns out that several regimes (zones) should be distinguished. For each regime we provide asymptotics for large deviation probability and give appropriate interpretation in terms of the service system.

At the end, we briefly discuss multivariate extensions of the model which admit different practical interpretations such as pollution model, internal bone structure model, etc.

This is a joint work with S.E. Nikitin. The results are published in [1] and [2].

References

- [1] M.A. Lifshits, S.E. Nikitin, *Large deviations of Telecom process.* J. Appl. Probab. **60** (2023), pp. 267–283.
- [2] M.A. Lifshits, S.E. Nikitin, *Ultralarge deviations of Telecom process.* Zap. Nauchn. Semin. POMI, **515** (2022), pp. 162–179 (Rus.).

Distances between random points inside and on the boundary of a convex body

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Kingman’s classical formula [1] from integral geometry establishes a relation between the moments of the mean distance between two random points chosen independently and uniformly inside a convex body in $\mathbb{R}^n$, and the moments of the length of its random section. We obtain an analogue of this formula in the case when one of the points is chosen at random on the boundary of the body. As a corollary, a relation between the distribution functions of the considered quantities is derived. An analogous result for two points inside the body was obtained by Moseeva [2].

The proof is based on the introduction of a new “homothetic” coordinate system adapted to the geometry of K , combining ideas of stochastic geometry and the theory of convex bodies.

References

- [1] J. Kingman, *Random secants of a convex body.* J. Appl. Probab. **6**, No. 3 (1969), 660–672.
- [2] T. Moseeva, *Random sections of convex bodies.* Probability and Statistics, 28, Zap. Nauchn. Semin. POMI, **486**, POMI, St. Petersburg, 2019, 190–199.

Random walk of a countable particle system with accumulation

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Consider a system of particles in the strip $\Pi = \mathbb{Z} \times \{0, 1, 2, \dots, N - 1\}$. At the initial moment of time at each point of the subset $\Pi_+ = \mathbb{N} \times \{0, 1, 2, \dots, N - 1\}$ there is exactly one particle. The dynamics of each particle is determined by a homogeneous and discrete in time, irreducible and non-periodic Markov chain $\xi_t = (\xi_t^1, \xi_t^2)$ with state space Π and with transition probabilities

$$p_{l_1 j_1}^{lj} = P(\xi_{t+1} = (l, j) | \xi_t = (l_1, j_1)), (l, j), (l_1, j_1) \in \Pi,$$

satisfying the conditions:

$$p_{l_1 j_1}^{lj} = 0 \text{ for } |l - l_1| > 1$$

and

$$p_{l_1 j_1}^{lj} = p_{l_1 - l, j_1}^{0j}.$$

It is assumed that the random walks of different particles are independent and identically distributed. When any particle first time enters the set

$$\Pi_0 = \{0\} \times \{0, 1, 2, \dots, N - 1\} \subset \Pi,$$

it is absorbed into it. Let $n(t)$ denote the number of particles absorbed (accumulated) in the set Π_0 by time t .

We define an ergodic Markov chain with the set of states $\{0, 1, 2, \dots, N - 1\}$ and with transition probabilities

$$q_{j_1}^j = p_{l+1 j_1}^{lj} + p_{l j_1}^{lj} + p_{l-1 j_1}^{lj}.$$

Let π_j denote the stationary distribution of this chain.

We set $v = \sum_{j=0}^{N-1} \pi_j m_j$, where $m_j = E(\xi_{t+1}^1 - \xi_t^1 | \xi_t^1 = 0, \xi_t^2 = j)$, $j \in \{0, 1, 2, \dots, N - 1\}$.

Theorem.

1) For $v < 0$ there exists $\frac{n(t)}{t} \xrightarrow[t \rightarrow \infty]{P} -Nv$, $t \rightarrow \infty$.

- 2) For $v > 0$ there exists a constant C such that $En(t) \leq C$.
- 3) For $v = 0$ there exists a limit $\frac{En(t)}{\sqrt{t}} \xrightarrow[t \rightarrow \infty]{} c$, for which the constant $c > 0$ can be found explicitly.

References

- [1] V.A. Malyshev, *Stochastic Growth Models without Classical Branching Processes*. Markov Processes and Related Fields **28** (2022), pp. 179–184.
- [2] G. Fayolle, V.A.Malyshev, M.V.Menshikov, *Topics in the Constructive Theory of Countable Markov Chains*. Oxford university press (1995).

On a combinatorial class of random walks

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For an arbitrary sequence s_i of length n , the greatest convex minorant is a greatest convex sequence h_i such that $h_i \leq s_i$. Its vertices are at the points where equalities are attained, and the edges are between them. For a standard random walk with independent identically distributed steps, the edge lengths of a convex minorant are distributed as cycles of an uniformly chosen permutation of size n .

We will consider another measure on random permutations, in which the cyclic type will be directly specified by the parameter θ . Using the generalized Vervaat transform, we will obtain a new random walk and consider the corresponding limit random process.

Random valuations

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Random valuation is a stochastic process Φ indexed by convex bodies in $\mathbb{R}^d$, which is also an additive function, meaning that

$$\Phi(K \cup L) + \Phi(K \cap L) = \Phi(K) + \Phi(L)$$

for all convex bodies K and L such that their union is also convex. If the sample paths of Φ are invariant under rigid motions and continuous, then Hadwiger's theorem immediately yields that Φ is a linear combination of intrinsic volumes with random coefficients. We aim to replace the motion invariance of the paths by distributional invariance properties of Φ , most importantly, stationarity.

In order to handle valuations without imposing motion invariance on their realisations, we first discuss what is possible to say about deterministic valuations which are not translation invariant, replacing the invariance property by other conditions, e.g., assuming that the valuations are integer-valued. Then we pass to the random setting and show that meaningful results on random valuations are possible, assuming their infinite divisibility under addition and a kind of independence of the increments together with a suitable continuity property. Under these assumptions and assuming stationarity and non-negativity, Φ is the sum of indicator random valuations $t_i \mathbf{1}_{K \cap F_i}$, where $\{(F_i, t_i), i \geq 1\}$ is a Poisson process on the product of the family of convex closed sets and the positive half-line.

The talk is based on joint work with Andrii Iliencko and Tommaso Visoná (Bern).

Inequalities for expected volumes of random polytopes

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The classical result of Miles [1] calculates moments of the volume of a random simplex, with some of vertices chosen uniformly inside the unit ball, and some — uniformly on the unit sphere. Miles' result implies that for a fixed number of vertices, the mean volume of the simplex increases as the number of vertices on the unit sphere increases.

The uniform distribution inside or on the boundary of the unit ball can be considered as a special case of the beta distribution: in the former case, the parameter is 0, while the uniform distribution on the unit sphere is obtained as a weak limit of beta distributions as the parameter tends to -1 .

Consider a random simplex where some vertices are chosen according to a beta distribution with parameter β_1 , and some with parameter $\beta_2 < \beta_1$. It turns out that the more vertices are chosen according to the second distribution (for a fixed total number of vertices), the larger the mean volume of the simplex is.

The above result can be generalized to random beta polytopes, which are not necessarily simplices. Furthermore, more general conditions can be introduced for the distributions of the vertices of two random polytopes, under which similar inequalities hold for their mean volumes, as well as for their mean intrinsic volumes. The talk is devoted to these generalizations.

References

- [1] R.E. Miles, *Isotropic random simplices*. Advances in Applied Probability **3** (1971), pp. 353 – 383.

Duality of Energy and Probability in Finite-Volume Models of Statistical Physics

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It is well-known that the Gibbs formula (which establishes a relationship between probability and energy) is the basis of statistical physics. Much attention has been paid to the justification of the Gibbs formula using physical reasoning. In [1], it was shown that the Gibbs formula can have a purely mathematical justification for both finite and infinite systems. This became possible thanks to the introduced concept of transition energy field.

In this talk, we will show that there is a deeper relationship between energy and probability, namely, in finite-volume models of statistical physics, energy and probability are dual concepts.

Duality in mathematics is the principle according to which any true statement of one theory corresponds to a true statement in the dual theory. Here, we will show how this principle can be applied to solve the known problem of describing a finite random field by a set of consistent conditional distributions (see, for example, [2]). A direct probabilistic solution to this problem was given in [3].

The talk is based on the joint work [4] with Linda Khachatryan, Institute of Mathematics, NAS RA.

References

- [1] S. Dachian and B.S. Nahapetian, *On the relationship of energy and probability in models of classical statistical physics*. Markov Processes Relat. Fields **25** (2019), pp. 649–681.
- [2] S. Geman and D. Geman, *Stochastic relaxation, Gibbs distributions, and the Bayesian restoration of images*. IEEE Transactions on Pattern Analysis and Machine Intelligence, **PAMI-6** (1984), pp. 721–741.
- [3] L.A. Khachatryan and B.S. Nahapetian, *On the characterization of a finite random field by conditional distribution and its Gibbs form*. J. Theor. Probab. **36** (2023), pp. 1743–1761.
- [4] L.A. Khachatryan and B.S. Nahapetian, *Duality of energy and probability in*

Tomography problems of Stochastic Geometry

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In the paper [1] an explicit formula for orientation-dependent chord length distribution function for any bounded convex domains $\mathbb{D}$ have been obtained (see also [2]). One can show that the problem of determining a body from this data, gave a positive answer using “orientation dependent” chord length distribution, when $\mathbb{D}$ is a planar convex polygon (see [3]). A set of four directions whose slopes have a transcendental cross ratio will ensure that the corresponding parallel X -rays determine each planar convex domain (see [3]). The following problem arise.

Does there exist a finite set of directions $V = \{\phi_1, \dots, \phi_m\}$ such that the corresponding set of “orientation-dependent” chord length distribution functions $F_{\mathbb{D}}(\phi_1, y), \dots, F_{\mathbb{D}}(\phi_m, y)$ determine a bounded convex domain uniquely. This question received negative answer, because it is possible to construct two non-congruent triangles that have the same chord length distribution function for a fixed set of m directions, where m is a natural (see [4]). The question arises whether it is possible to find a subclass of convex bodies, where it is possible to reconstruct a body from the values of $F_{\mathbb{D}}(\phi, y)$ for a finite set of directions.

Proposition ([5]. Let $\mathbb{D}$ be a convex planar polygon which has m pairs of parallel sides $(a_{i_1}, a_{j_1}), \dots, (a_{i_m}, a_{j_m})$. The distances of the parallel lines which carry these segments are $d_1, \dots, d_m$, respectively, and $\pi a_{i_k} \cap \pi a_{j_k}$ denotes the length of the intersection of the orthogonal projections of both segments onto one of the carrying lines, $k = 1, \dots, m$. Then for $k \in \{1, \dots, m\}$ for which $\pi a_{i_k} \cap \pi a_{j_k} > 0$, the chord length density function has a discontinuity at d_k , and the limit from above at d_k is infinite.

The concept of covariogram is extended from bounded convex bodies in $\mathbf{R}^d$ to

the entire space $\mathbf{R}^d$ by obtaining integral representations for the distribution and density functions of the Euclidean distance between two d -dimensional Gaussian points that have correlated coordinates governed by a covariance matrix. When $d = 2$, a closed-form expression for the density function is obtained. Precise bounds for the moments of the considered distance are found in terms of the extreme eigenvalues of the covariance matrix (see [5]).

References

- [1] N. G. Aharonyan *Generalized Pleijel Identity*. Journal of Contemporary Mathematical Analysis **43**, no. 5, 2008, pp.3–12.
- [2] R. V. Ambartzumian *Factorization Calculus and Geometric Probability*, Cambridge University Press, Cambridge, 1990.
- [3] R. J. Gardner (2006) *Geometric Tomography*. 2nd edn, Cambridge University Press, Cambridge, UK, New York. 2006.
- [4] A. Gasparyan, V.K. Ohanyan *Recognition of triangles by covariogram*. Journal of Contemporary Mathematical Analysis **48**, no. 3, 2013, pp. 110 – 122.
- [5] V. K. Ohanyan and D. M. Martirosyan *On intersection probabilities of four lines inside a planar convex domain*. Journal of Advances in Applied Probability, London, **60**, no. 2, 2023, pp. 416–430.
- [6] D. M. Martirosyan and V. K. Ohanyan *On the Euclidean Distance Between Two Gaussian Points and the Normal Covariogram of $\mathbf{R}^d$* . Journal of Contemporary Mathematical Analysis, **59**, no 1, 2024, pp. 38-46.

Finding the Volume, Surface Area, and Edge Length Distributions for Poisson Plane Processes

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We present a three dimensional analogue of the moving secant method developed in the planar case by Belov et al. [2]: a random polytope is probed by a moving plane, while we track only a few on section variables (a secant segment with its two endpoint angles together with the running section area), plus the accumulated

volume V , surface area A , and total edge length U . This reduction yields finite dimensional kinetic (transport) equations on the plane; exact projection relations using the Crofton cosine weight over plane orientations lift the along plane laws to spatial (3D) distributions. The same kinetic projection pipeline applies, with the same state variables, to the typical cell of the three dimensional Poisson Voronoi diagram. In both models a single system of equations delivers the global (“total”) distributions aggregated over all combinatorial types so no enumeration by face edge vertex counts is required. Laplace transforms of the kinetic system yield closed forms in some cases and integral representations in general.

Miles [3] develops the general theory of random partitions of $\mathbb{R}^d$ by Poisson flats, deriving exact intensities and mean geometric characteristics of cells, faces, edges, and vertices via integral geometric balance relations. Our approach draws on classical integral geometric and stereological foundations. It extends the planar moving line framework of Belov et al. [2]. Its scope is different from work centered on combinatorial invariants like angles and face numbers of random polytopes (Kabluchko, [1]); here we target the size characteristics and their joint laws directly, without conditioning on the number of faces. Beyond the Poisson plane and Poisson Voronoi models, the moving plane kinetic scheme extends (often with minor changes) to other Crofton type problems in spatial integral geometry, providing a reusable route to full distributions without combinatorial indexing.

The talk is based on joint work with Mehdi Golafshan.

References

- [1] Z. Kabluchko, *Angles of random simplices and face numbers of random polytopes*. Advances in Mathematics **380** (2021), 107612.
- [2] A.Ya. Kanel–Belov, M. Golafshan, S.G. Malev, and R.P. Yavich, *Finding the Area and Perimeter Distributions for Flat Poisson Processes of a Straight Line and Voronoi Diagrams*. Doklady Mathematics (2024).
- [3] R.E. Miles, *The random division of space*. Advances in Applied Probability **4** (1972), pp. 243–266.

Limit Theorems for Classical Gases

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We consider classical continuous system of interacting identical particles in Euclidean space $\mathbb{R}^d$ (classical gas). Our approach to the limit theorems for the particle number is based on the method of cluster expansions which is well known for finite Gibbs processes. In case of the limiting Gibbs process with empty boundary conditions, we use our result on the cluster representation of such processes which goes back to Malyshev and Minlos. We prove integral and local central limit theorems for a large class of stable and regular pair potentials which include physically relevant interactions if the activity is small. In case of local central limit theorem we obtain a second order asymptotic expansion for the number of particles in bounded region A as $A \uparrow \mathbb{R}^d$.

The talk is based on joint work with Hans Zessin.

References

- [1] Poghosyan, S., Zessin, H.: Once more the Central Limit Theorem for the particle number of a Classical Gas, to appear in the Annales de la Faculté des Sciences de Toulouse.
- [2] Poghosyan, S., Zessin, H.: Central Limit Theorem for the particle number of Classical Gases. Local approach, Journal of Mathematical Physics, 66, (053306), (2025)
- [3] Poghosyan, S., Zessin, H.: Second order asymptotic expansion in the local Limit Theorem for the particle number of a Classical Gas, submitted to the Annales Henri Poincaré.

Estimation of negative sectional curvature from metric treeness of uniform samples

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We consider a problem of statistical estimation of constant sectional curvature of spaces of various dimensions – from finite uniform samples of points from balls of various radii in these geometries – only using information on true pairwise distances between the points. For that, we develop a computational method of generating various such discrete metric d samples, and study the distribution of the following statistic:

$$\gamma(x, y) = d_T(x, y) - d(x, y) \quad (6)$$

for all pairs (x, y) of $N \gg 1$ points, where d is true distance, and d_T is its “rough tree approximation” – namely, length of a path in the minimal spanning tree (MST) of the initial complete metric graph.

Compared to Fournier et al., who show [1] that supremum of γ -s can be used to effectively (albeit with a $2 \log_2 N$ -multiplicative error) estimate the underlying Gromov’s delta-hyperbolicity δ , we study the full distribution of γ statistics for such uniform metric ball samples.

Our work is heavily inspired by potential applications of such estimation in the recently developing field [2-3] of machine learning with hyperbolic embeddings (widely believed well applicable for hierarchical data structures); in particular for recommender systems [3], where effective estimation of data hyperbolicity has proven useful.

References

- [1] H. Fournier, A. Ismail, A. Vigneron. *Computing the Gromov hyperbolicity of a discrete metric space*. Information Processing Letters **115**(6-8) (2015), pp. 576-579.
- [2] W. Peng, T. Varanka, A. Mostafa, H. Shi, G. Zhao. *Hyperbolic deep neural networks: A survey*. IEEE Transactions on pattern analysis and machine intelligence **44.12** (2021), pp. 10023-10044.

[3] E. Frolov, T. Matveeva, L. Mirvakhabova, I. Oseledets. *Self-Attentive Sequential Recommendations with Hyperbolic Representations*. Proceedings of the 18th ACM Conference on Recommender Systems. (2024), pp 981–986.

On the Mean Number of Convex Records for Light-Tailed Distributions

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Assume that the random variables $X_1, X_2, \dots$ are independent and identically distributed. We say that X_n is a record if it is larger than all of its predecessors $X_1, \dots, X_{n-1}$. For random vectors in higher-dimensional spaces, there are numerous generalizations of this concept, most of which rely in some way on the vectors' coordinates. In this talk, we will focus on convex records — a generalization defined through a geometric approach. Here, a vector X_n is called a convex record if it lies outside the convex hull of the previous points $X_1, \dots, X_{n-1}$.

I will discuss several asymptotic properties of these convex records. Specifically, I will present results on the asymptotic behavior of the expected number of records that appear near the convex hull boundary. The emphasis will be on the case of random vectors in the plane whose distributions have symmetric tails that decay exponentially.

Random eigenvalues of fullerenes, graphene and nanotubes

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The spectral analysis of carbon allotropes has recently emerged as a unifying theme at the intersection of convex geometry, combinatorics, probability theory, and mathematical physics. In this talk, we present a series of recent advances on the study of *random eigenvalues* of lattices and fullerene-type combinatorial graphs.

We first discuss explicit formulas for the probability density and characteristic functions of random eigenvalues of the hexagonal (graphene) lattice and its dual triangular lattice. These formulas, derived via novel integral identities for cubic powers of modified Bessel functions, reveal deep connections between spectral distributions, closed walks, and planar random flights. An efficient simulation method based on uniform random variables will also be highlighted.

We then turn to infinite nanotubes, modeled mathematically as “rolled-up” hexagonal lattices determined by a chiral vector (p, q) . For these structures, we show that the sequence of closed walk counts forms a moment sequence of non-linear functions of two independent uniform random variables. Explicit density and moment generating functions are obtained for special classes of nanotubes, and we prove that as the circumference $p + q \rightarrow \infty$, their spectral distributions converge weakly to that of graphene.

Finally, if time permits, we address recent some open problems on fullerenes. Using local weak convergence, we conjecture that large random fullerenes approximate the spectral distribution of graphene, thus linking finite carbon cages to infinite lattices. Several combinatorial and probabilistic challenges remain open, including the precise characterization of the limiting empirical spectral distributions and the structural role of pentagons in fullerene spectra. Another challenging subject is that our spectra for infinite nanotubes seem to coincide with the approximate solutions to Schrödinger equations for molecular energy of carbon nanotubes.

This is joint work with A. Bille, V. Buchstaber, S. Coste, P. Ievlev, S. Kuriki and S. Novikov.

References

- [1] A. Bille, V. Buchstaber, P. Ievlev, S. Novikov, E. Spodarev, *Random eigenvalues of nanotubes*. J. Phys. A: Math. Theor. **58** (2025), 105202.
- [2] A. Bille, V. Buchstaber, S. Coste, S. Kuriki, E. Spodarev, *Random eigenvalues of graphenes and the triangulation of plane*. J. Phys. A: Math. Theor. **58** (2025), 105202.
- [3] A. Bille, V. Buchstaber, E. Spodarev, *Some open mathematical problems on fullerenes*. Journal of Chemical Information and Modeling **65** No. 6 (2025), p. 2911-2923.

Inequalities in Stereology and Their Application to Engineering Problems

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Stereology studies the properties of three-dimensional objects based on their two-dimensional sections by random planes. The distribution of random planes is generated by the measure invariant with respect to the group of Euclidean motions in R^3 . This measure is unique (up to a constant factor) and its element has the form $\mu(de) = dp d\omega$, where p is the distance from the origin to plane e , ω is the spatial direction of the normal to e . We study the probability distribution

$$P_n = \frac{\mu(A_n)}{\mu([D])},$$

where D is a convex polyhedron, A_n is the event that the section $e \cap D$ is an n -gon, $[D]$ is the set of planes intersecting D .

Families of polyhedra containing a regular polyhedron (tetrahedron, cube, octahedron) have been studied. A theorem has been proven that for such families the probabilities P_n reach their extremum on a regular polyhedron. Using this

theorem some inequalities for probabilities P_n are obtained.

It is shown how these inequalities can be used to solve the practical engineering problems.

Lattice points inside a randomly shifted integer polyhedron

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Consider a convex body $C \subset \mathbb{R}^d$. Let X be a random point with a uniform distribution in $[0, 1]^d$. Define X_C as the number of lattice points in $\mathbb{Z}^d$ inside the translated body $C + X$. It is well known that $\mathbb{E}X_C = \text{vol}(C)$. A natural question arises: What can be said about the distribution of X_C in general? In this presentation, we study this question when C is a polyhedron with vertices at integer points. In the 2-dimensional case, we show that the variance depends only on the number of points on the sides of the polygon. Moreover, we derive an explicit formula for the distribution of X_P when P is an integer triangle. In higher dimensions, we present generalizations of several two-dimensional results, as well as provide counterexamples to other statements.

Records and geometry distribution

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Consider a sequence of independent identically distributed random variables $X_1, X_2, \dots$ taking values in $\mathbb{N}$ with common law X whose distribution has no

finite right endpoint: $\mathbb{P}(X < n) < 1$ for all $n \in \mathbb{N}$. Define the *strict (upper) record times* recursively by

$$T_0 = 1, \quad T_{n+1} = \inf\{j > T_n : X_j > X_{T_n}\}, \quad n \geq 0,$$

and the corresponding *record values* $R_n := X_{T_n}$, $n \geq 1$. For $\beta \in (0, 1)$ denote by $A_n(\beta)$ the distribution of R_n in the case where the parent variable X has the geometric distribution $\text{Geom}(\beta)$ on $\mathbb{N}$, that is,

$$\mathbb{P}(X = k) = \beta(1 - \beta)^{k-1}, \quad k \in \mathbb{N}.$$

Two main results are obtained.

Theorem 1 (two-record characterization of the geometric distribution). If for some X one has

$$R_1(X) \stackrel{d}{=} A_1(\beta_1) \quad \text{and} \quad R_2(X) \stackrel{d}{=} A_2(\beta_2),$$

then $\beta_1 = \beta_2$, and moreover $X \stackrel{d}{=} \text{Geom}(\beta_1)$.

Theorem 2 (class of distributions with a fixed first strict record). We give a complete (parametric) description of the family of distributions X on $\mathbb{N}$ for which

$$R_1(X) \stackrel{d}{=} A_1(\beta) \quad \text{for a fixed } \beta \in (0, 1).$$